

Single j -shell studies of cross-conjugate nuclei and isomerism: $(2j - 1)$ rule

L. Zamick* and A. Escuderos

Department of Physics and Astronomy, Rutgers University, New Brunswick, NJ 08903, USA

Isomeric states for 4 nucleons with isospin $T = 1$ are here considered. A comparison is made of the lighter and heavier members of cross-conjugate pairs where one member is obtained from the other by replacing protons by neutron holes and neutrons by proton holes. Although in the single j -shell the spectra in the two cases should be identical, this is not the case experimentally. For the former, the ground states all have angular momentum $J = 2$. This result is found in a single j -shell calculation when the interaction is obtained from the spectrum of two particles. In a single j -shell ($f_{7/2}$, $g_{9/2}$) the state with angular momentum $(2j - 1)$ is the ground state for the heavier member of the pair provided one uses as an interaction the spectrum of two holes. The ground state behaviour can also be explained by rotational models. A new observation is that both in single j and in experiment the $J = 2$ state in the heavier member and the $(2j - 1)$ state in the lighter member are isomeric.

PACS numbers: 21.60.Cs

I. INTRODUCTION

In this work we develop a rule based on interesting behaviours of nuclear spectra, or to be more precise spectra of four-nucleon states with isospin $T = 1$ in odd-odd nuclei. Such states consist of either three protons and one neutron or three neutrons and one proton; also three proton holes and one neutron hole or three neutron holes and one proton hole. We invoke the single j -shell concept of cross-conjugate (CC) pairs. We reach a CC partner by replacing valence neutrons by proton holes and valence protons by neutron holes, e.g. ^{44}Sc consists of one proton and three neutrons in the $f_{7/2}$ shell, whilst its CC partner has one neutron hole (seven neutrons) and three proton holes (five protons)—ergo, ^{52}Mn . If one uses the same interaction to calculate the energy levels of these pairs in the single j -shell model space, one obtains identical spectra for the CC partners.

We find in single j -shell calculations that in the $f_{7/2}$ and $g_{9/2}$ shells, states with total angular momentum $J = (2j - 1)$ (which is the same as median angular momentum $J = (J_{\max} + 1)/2$) lie low in energy and become isomeric for lighter members of cross-conjugate pairs and ground states for the heavier members. Conversely, $J = 2^+$ states are ground states for the lighter members and isomeric for the heavier members. Although these calculations are relatively simple—not large scale—, they are supported by experiment. We note that J_{\max} is equal to M_{\max} . For three neutrons the maximum value of M is $j + (j - 1) + (j - 2)$ and for the single proton it is j . Thus J_{\max} is equal to $(4j - 3)$ whilst $(J_{\max} + 1)/2$ is equal to $(2j - 1)$. To briefly summarise the findings, we note that for the two shells listed above the values of J_{\max} are 11 and 15, respectively. Thus the $(2j - 1)$ rule gives values of 6 and 8 for the low-lying isomeric (or ground) states. We emphasize that the single j -shell

model is used only to make qualitative statements about isomerism.

II. THE $f_{7/2}$ SHELL

We start with the $f_{7/2}$ shell where single j -shell calculations have already been performed and wave functions tabulated by Zamick, Escuderos, and Bayman [1]; this reference is based on previous work of Refs. [2–4]. The interaction used consists of matrix elements taken from experiment—more precisely from the spectrum of ^{42}Sc and ^{42}Ca (INTa). Zamick, however, noted that for the upper half of the $f_{7/2}$ shell one obtains better results by using matrix elements from the two-hole system ^{54}Co (INTb) [5]. In single j -shell calculations with both neutrons and protons, we define the cross-conjugate of a given nucleus as one in which protons are replaced by neutron holes and neutrons by proton holes. Thus ^{52}Fe is the cross-conjugate of ^{44}Ti and ^{52}Mn is the cross-conjugate of ^{44}Sc . If one uses the same charge independent two-body interaction in both nuclei, the spectra for states in this limited model space should be identical. In fact, although the spectra are similar, they are not identical experimentally. The 10^+ state in ^{44}Ti is below the 12^+ , but in ^{52}Fe the reverse is true. In both cases the 12^+ state is isomeric but the one in ^{52}Fe has a much longer half-life because it cannot decay to the 10^+ state. As seen in Table I we are successful in getting the 12^+ below the 10^+ by using the spectrum of ^{54}Co as input. The main difference in the two-body spectra is that the $J = 7^+$ state in ^{54}Co is much lower in energy than it is in ^{42}Sc (see Table VII).

Large space shell-model calculations for ^{52}Fe were performed by Ur *et al.* [6] using the KB3 interaction and by Puddu [7] using the GXPF1A interaction. Both groups get a near degeneracy of 10_1^+ and 12_1^+ in ^{52}Fe . Thus, although they do not get 12^+ sufficiently below 10^+ , they do go in the right direction relative to ^{44}Ti . Ur *et al.* attribute increased collectivity in ^{52}Fe mainly to $p_{3/2}$ ad-

*Electronic address: lzamick@physics.rutgers.edu

Table I: Yrast spectra of ^{44}Ti and ^{52}Fe calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

$E(\text{MeV})$				
^{44}Ti			^{52}Fe	
J	INTa	Exp.	INTb	Exp.
0	0.000	0.000	0.000	0.000
1	5.669		5.442	
2	1.163	1.083	1.015	0.849
3	5.786		5.834	
4	2.790	2.454	2.628	2.384
5	5.871		6.463	
6	4.062	4.015	4.078	4.325
7	6.043		5.890	
8	6.084	(6.509)	5.772	6.361
9	7.984		7.791	
10	7.384	(7.671)	6.721	7.382
11	9.865		8.666	
12	7.702	(8.040)	6.514	6.958

mixtures for the reason there are differences in the cross-conjugate pairs.

We then examine the yrast spectrum of ^{44}Sc calculated with the interaction INTa (see Table II). We consider two groups. First for $J=6, 5, 4, 3, 2$, and 1, the energies in MeV are respectively 0.38, 1.28, 0.71, 0.76, 0.00, and 0.43 (the $J=0^+$ state has isospin $T=2$ and is at an excitation energy of 3.047 MeV). We see that the only state below the $J=6$ state is $J=2$. Thus, the lowest multipolarity for decay is $E4$ and so the $J=6$ state is calculated to be isomeric. For the second group with $J=11, 10, 9, 8$, and 7, the energies in MeV are respectively 4.64, 4.79, 3.39, 3.10, and 1.27. The $J^\pi=11^+$ state can decay via an $E2$ transition to the $J^\pi=9^+$ state so it should not be isomeric.

Table II: Yrast spectra of ^{44}Sc and ^{52}Mn calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

$E(\text{MeV})$				
^{44}Sc			^{52}Mn	
J	INTa	Exp.	INTb	Exp.
0	3.047		2.774	
1	0.432	0.667	0.443	0.546
2	0.000	0.000	0.202	0.378
3	0.764	0.762	0.836	0.825
4	0.713	0.350	0.851	0.732
5	1.276	1.513	1.404	1.254
6	0.381	0.271	0.000	0.000
7	1.272	0.968	1.819	0.870
8	3.097		2.572	(2.286)
9	3.390	2.672	2.792	(2.908)
10	4.793	4.114	4.365	4.164
11	4.638	3.567	3.667	(3.837)

We now look at experiment. In ^{44}Sc the lowest $J^\pi=6^+$ state has a half-life of 58.6 hours—it is indeed isomeric.

But we should also consider the cross-conjugate nucleus ^{52}Mn consisting of three proton holes and one neutron hole relative to ^{56}Ni . We see that here the $J^\pi=6^+$ state is the ground state with a half-life of 5.591 days. As mentioned before, if we use the same interaction here as we did for ^{44}Sc , we would not get the $J=6^+$ state as the ground state. But as seen in Table II, when we use as input the spectrum of the two-hole system ^{54}Co , we do get $J=6^+$ as the ground state.

There is some indication that in heavier nuclei the state with $J=J_{\text{max}}$ should be isomeric. However, the $J^\pi=11^+$ state at 3.57 MeV in ^{44}Sc has a half-life of 48 ps whilst the corresponding $J^\pi=11^+$ state in ^{52}Mn at 3.84 MeV has a half-life of 15.1 ps.

We could not find large-space shell-model calculations of ^{52}Mn in the literature, but there is a single j -shell calculation in the work of Avrigeanu *et al.* [9]. This accompanies their experimental work on high-spin states in this nucleus.

III. THE $g_{9/2}$ SHELL

In previous work [10], calculations were performed in the $g_{9/2}$ shell where the emphasis was on partial dynamical symmetries. It was there noted that the single j -shell model for $g_{9/2}$ works well for proton holes relative to $Z=50, N=50$ but not for neutrons relative to $Z=40, N=40$. We will therefore focus on the region near $Z=50, N=50$.

Nara Singh [11] reported the finding by his group of a $J^\pi=16^+$ isomeric state in ^{96}Cd that beta decayed to a $J^\pi=15^+$ state in ^{96}Ag , which is also isomeric. This largely stimulated the work done here on isomerism. We also note a combination of experiment and shell-model calculations by K. Schmidt *et al.* [12] and L. Batist *et al.* [13]. The topics addressed in these works are decay properties of very neutron-deficient isotopes of silver and cadmium, as well as isomerism in ^{96}Ag .

We show results for two interactions: INTc and INTd (see Table III). The $T=1$ matrix elements are obtained from the spectrum of ^{98}Cd , that is, two proton holes. Unfortunately, the spectrum of ^{98}In is not known, so we cannot get the $T=0$ matrix elements from experiment. We use a delta interaction to generate the $T=0$ matrix elements for INTc. Noting that in the $f_{7/2}$ shell the state with $J=J_{\text{max}}$, i.e. $J=7$, comes much lower for two holes than it does for two particles, we simulate this behaviour in INTd in the $g_{9/2}$ shell by changing the $J_{\text{max}}=9$ energy from 1.4964 MeV to 0.7500 MeV, leaving all other two-body matrix elements the same. This interaction should be more appropriate for the four-hole system.

With the INTc interaction, the $J=2^+$ state is the ground state and should be long-lived. The $J=8^+$ is at an excitation energy of 0.350 MeV, so only the $J=0^+$ ($T=2$) and $J=2^+$ states are below it. So this state should be isomeric. But for INTd, where we lowered

Table III: Energy levels for the case of 3 protons and 1 neutron in the $g_{9/2}$ shell with the interactions INTc and INTd (see text), and compared with the experimental data for ^{96}Ag .

J	$E(\text{MeV})$		
	INTc	INTd	Exp.
0	0.246	0.900	
1	0.463	0.483	
2	0.000	0.097	
3	0.638	0.588	
4	0.394	0.349	
5	0.774	0.737	
6	0.450	0.371	
7	0.850	0.861	
8	0.350	0.000	0.000
9	0.872	0.492	0.470
10	2.188	1.748	(1.719)
11	2.344	1.930	(1.976)
12	3.004	2.550	
13	3.087	2.556	2.643
14	3.382	3.070	
15	3.287	2.645	2.643+x

the energy of the $J = 9^+$ two-body matrix element, the $J = 8^+$ state is now the ground state and is of course long lived. The $J = 2^+$ state is very low lying (0.097 MeV) and is isomeric. At high spin with INTd the $J = 15^+$ state is at 2.645 MeV while the $J = 13^+$ state is at 2.556 MeV. Because they are so close in energy, the $J = 15^+$ state is isomeric.

Concerning the experiments in Refs. [12, 13], nearly degenerate $J^\pi = 2^+$ and $J^\pi = 8^+$ lowest lying states are shown with respective half-lives of 6.9(6) s and 4.40(6) s. We see that also in this shell the $(2j - 1)$ rule is verified.

We find that, unlike in the $f_{7/2}$ shell, here in $g_{9/2}$ our calculation with INTd leads to an isomeric state for $J = J_{\text{max}} = 15$ and this supports the experimental findings of Nara Singh [11]. We now refer to the experimental works of Grzywacz *et al.* [14] and Grawe *et al.* [15]. The latter work also includes large-scale shell-model calculations and points out that there are many spin-gap states in the ^{100}Sn region. A near degeneracy of the two states in ^{96}Ag is shown in Fig. 1 of Grawe *et al.*, with the $J = 13^+$ state indeed ever so slightly below the $J = 15^+$ state.

IV. EXPLANATIONS OF THE ISOMERISMS

We have admittedly done some very simple calculations, but that is the point. One should do such calculations to search for interesting behaviours. Later one can supplement these with more detailed calculations. The simple calculations are useful when effects are large as in the case of the $(2j - 1)$ rule.

The ground states of odd-odd nuclei have been considered by Gallagher-Moszkowski [16]. They developed a scheme for obtaining and predicting the ground state

spins of odd-odd nuclei. Briefly stated, the value of the total angular momentum is predicted to be $(\Omega_p + \Omega_n)$ in some cases and $|\Omega_n - \Omega_p|$ in others, where Ω is the component of the angular momentum along the symmetry axis. In the weak deformation limit, one gets the plus sign if $j = L + 1/2$ for both protons and neutrons, and the minus sign if $j = L + 1/2$ for the neutrons and $j = L - 1/2$ for the protons or vice versa. In the cases we consider, we have $j = L + 1/2$ for both neutrons and protons, so the plus sign is appropriate.

Assuming a prolate deformation for a system of three neutrons and one proton, the Ω values are 1/2 for the proton and 3/2 for the three neutrons—hence $\Omega = 2$. We assume this is a band head for a rotational band and equate the laboratory angular momentum J with the intrinsic quantum number Ω . In ^{96}Ag we have one neutron hole with $\Omega_n = j$ and three proton holes with $\Omega_p = j - j + (j - 1) = (j - 1)$; hence, $\Omega = (2j - 1)$. We then have for the ground state $J = \Omega = (2j - 1)$. Note that $(2j - 1)$ is also $(J_{\text{max}} + 1)/2$, where J_{max} is the largest possible angular momentum for four nucleons in a j -shell with isospin $T = 1$.

To complete the argument, we note that in the single j -shell model a nucleus and its cross-conjugate partner should have identical spectra. This is not the case experimentally. The lighter members have $J = 2$ ground states and the heavier ones $J = (2j - 1)$ ground states. As far as the isomerism rule is concerned, we would argue that for the lighter members of the cross-conjugate pairs the shell effects are present, which, although not strong enough to maintain identical spectra with their partners, are nevertheless strong enough to keep the $(2j - 1)$ states sufficiently low as to be isomeric in the lighter members and the $J = 2^+$ states to be isomeric in the heavier ones.

V. ISOBARIC ANALOG STATES— $f_{7/2}$ VS. $g_{9/2}$

The $J = 0^+$ states in Tables II and III have isospins $T = 2$ while the other states have $T = 1$. The $J = 0^+$ states in ^{96}Ag are isobaric analog states of $J = 0^+$ states of the four proton-hole nucleus ^{96}Pd . Note that with the interactions that we have used, the $J = 0^+$ states lie much lower in the $g_{9/2}$ shell than in the $f_{7/2}$ shell, as far as a system of three protons and one neutron is concerned. There actually are two $T = 2$, $J = 0^+$ states for $(g_{9/2})^4$, only one for $f_{7/2}$. With INTd the lowest $J = 0^+$ state is at an excitation of 0.900 MeV, a prediction for ^{96}Ag . In ^{44}Sc and ^{52}Mn the excitation energies are 3.047 and 2.774 MeV respectively. Some caution must be used because of the uncertainty of the $T = 0$ two-body matrix elements in the $g_{9/2}$ shell.

VI. A BRIEF DISCUSSION OF HIGH-SPIN STATES IN ^{96}Cd

Three very closely timed publications have appeared on the subject of isomerism for $A = 96$. In reference [11] Nara Singh *et al.* first found a $J = 16^+$ isomeric state in ^{96}Cd . Indeed at the time of this writing this is the only known state in this nucleus. A recent work by A.D. Becerril *et al.* [17] is very relevant to the work discussed here. They find two isomeric states in ^{96}Ag . They do not assign spins but they are probably 15^+ and 13^- . Then there is the work of P. Boutachkov *et al.* [18] which follows from the findings of reference [11]. They observe the direct decay of the isomeric 16^+ state of ^{96}Cd to the 15^+ isomeric state in ^{96}Ag and are able to determine the spins of this and other isomers.

Our single j -shell calculation also yields a $J = 16^+$ isomer for ^{96}Cd (see Table IV). We see that the $J = 16^+$ state is calculated to be lower than $J = 15^+$ for INTc and lower than both $J = 15^+$ and 14^+ for INTd. This guarantees isomerism in this model space. In principle this could be upset by the appearance of negative parity states and electric dipole transitions but this does not seem to be the case experimentally.

Table IV: Calculated energies of states for ^{96}Cd from $J = 10^+$ to 16^+ .

J^π	$E(\text{MeV})$	
	INTc	INTd
10^+	4.570	4.617
11^+	5.312	5.564
12^+	5.232	5.630
13^+	5.696	5.895
14^+	5.430	5.030
15^+	6.625	5.564
16^+	5.506	4.937

VII. A RECENT $g_{9/2}$ INTERACTION—CCGI

While the current work was under consideration, a new interaction appeared in the literature. Coraggio *et al.* [19] developed an effective single j -shell interaction for the $g_{9/2}$ shell (we here call it CCGI) appropriate for nuclei close to ^{100}Sn . This is just what we need. They considered even-even and odd-even nuclei. We here apply their np two-body matrix elements to the odd-odd nucleus ^{96}Ag in order to test if our previous assertions are correct. Their two-body matrix elements are listed in appendix A and the calculated yrast spectra for ^{96}Cd (which they have already shown in their paper) and for ^{96}Ag are shown in Table V.

Of course, our main interest is whether the $J = (2j - 1)$ and $J = 2$ states are the lowest lying. Indeed they are. The $J = (2j - 1) = 8^+$ state is the ground state and the $J = 2^+$ state is the first excited state at 0.180 MeV. This

Table V: Yrast levels of ^{96}Cd and ^{96}Ag with the CCGI interaction (see text).

J	$E(\text{MeV})$	
	^{96}Cd	^{96}Ag
0	0.000	0.842 ($T = 2$)
1	4.269	0.449
2	1.081	0.180
3	4.467	0.648
4	2.110	0.338
5	4.556	0.746
6	2.888	0.286
7	4.635	0.815
8	3.230	0.000
9	4.365	0.545
10	4.881	1.959
11	5.913	2.214
12	5.339	2.666
13	6.107	2.663
14	5.403	3.099
15	6.550	2.731
16	5.245	

shows that the $J = (2j - 1), J = 2$ rule is reasonably robust.

VIII. RESULTS USING THE SPECTRUM OF ^{90}Nb

In the single j -shell model, ^{90}Nb consists of a $g_{9/2}$ neutron and a $g_{9/2}$ proton hole. The yrast spectrum of ^{90}Nb from $J = 0^+$ to $J = 9^+$ in MeV is: 5.008, 0.382, 0.854, 0.652, 0.328, 0.285, 0.122, 0.171, 0.000, and 0.812.

Sorlin and Porquet [20] use ^{90}Nb as input to obtain the particle-particle matrix elements. We can obtain the particle-particle spectrum from the particle-hole spectrum via the transformation:

$$V(pp, J) = - \sum_K (2K + 1) \begin{Bmatrix} 9/2 & 9/2 & K \\ 9/2 & 9/2 & J \end{Bmatrix} V(ph, K)$$

Thus, the resulting particle-particle spectrum from $J = 0^+$ to $J = 9^+$ is: -1.3032, -1.7947, -0.1809, -0.9841, 0.4915, -0.6050, 0.2784, -0.4791, 0.0298, -0.7462

In Table VI we can see the spectra of ^{96}Cd and ^{96}Ag obtained with this interaction (second and third columns, respectively). We observe a rather peculiar result: that the $J = 1^+$ particle-particle state is below the $J = 0^+$ state.

In the ^{96}Ag spectrum, now the lowest state is 1^+ . The $(2j - 1) = 8^+$ state, although not the ground state, is still isomeric.

We note that there are several low-lying 1^+ states in ^{90}Nb . The energies of the lowest four in MeV are 0.382, 1.344, 1.769, and 1.845. Undoubtedly the $g_{9/2}$ strength is at a higher energy than the yrast state. This makes the extraction of the particle-particle matrix elements

from the particle-hole spectrum complicated. But we explore this idea by making one replacement: we change the input 1^+ energy from that of the lowest 1^+ state at 0.382 MeV to that of the first excited state at 1.344 MeV. The resulting particle-particle spectrum from $J = 0^+$ to $J = 9^+$ in MeV is: -1.5918, -1.3420, -0.4390, -0.7654, 0.1372, -0.4913, 0.2346, -0.3647, 0.1610, -0.9823. We see that now the 0^+ state is the lowest state in the particle-particle spectrum.

Now for ^{96}Ag the ground state has $J = 2^+$ and the next lowest state is $J = 8^+$ at 0.033 MeV. This is more in accord with the previous analyses. Both would be long-lived states.

Table VI: Yrast levels of ^{96}Cd and ^{96}Ag calculated with two interactions obtained from the spectrum of ^{90}Nb (see text).

$E(\text{MeV})$				
$E(1^+) = 0.382 \text{ MeV}$		$E(1^+) = 1.344 \text{ MeV}$		
J	^{96}Cd	^{96}Ag	^{96}Cd	^{96}Ag
0	0.000	1.576	0.000	1.076
1	2.290	0.000	3.122	0.188
2	0.831	0.399	0.771	0.000
3	2.605	0.654	3.293	0.359
4	1.711	0.759	1.611	0.247
5	2.816	0.838	3.492	0.558
6	1.866	0.584	2.205	0.326
7	2.548	0.852	3.441	0.678
8	1.436	0.334	2.083	0.033
9	2.670	0.937	3.292	0.518
10	2.730	1.739	3.597	1.498
11	3.554	2.361	4.262	1.676
12	3.283	2.039	4.157	2.160
13	3.789	2.037	4.597	2.113
14	3.411	2.038	4.155	2.343
15	4.005	1.796	4.967	2.034
16	3.189		3.899	

IX. ADDED COMMENTS

We emphasize here that we are making only qualitative statements about isomerism, i.e. which angular momenta are and are not isomeric. We make comparisons of cross-conjugate pairs. Cross-conjugation is a single j -shell concept and so we invoke this model for insight into the behaviour of these four-nucleon $T = 1$ systems. We then note that memory of the single j -shell persists even in larger space calculations and indeed in nature. This explains the criss-cross behaviour so that $J = 2^+$ states in lighter members of a cross-conjugate pair are ground states and in the heavier members they are sufficiently low lying so as to be isomeric. Likewise $(2j - 1)$ states are sufficiently low lying so as to be isomeric for lighter members and ground states for heavier members. An important point in obtaining these results is that one should use as input the two-particle spectrum for the lighter member of the cross-conjugate pair and the two-

hole spectrum for the heavier pair. The most obvious difference is that the energy of the two-hole state with $J = J_{\text{max}} = 2j$ is much lower than the corresponding energy for two particles.

It should be further noted that the energy levels come out fairly well in the single j -shell model when compared with experiment (see Tables I, II, and III). Note that the sudden drop in the $J^\pi = 9^+$ energy of ^{96}Ag is correctly reproduced. This shows that the single j -shell model has considerable validity for the cases considered.

Most importantly we feel that after addressing the properties of a given nucleus, either theoretically or experimentally, one should try to see if the specific results are part of a bigger picture. This is certainly the case here. For example, the striking analogous behaviours in ^{52}Mn and ^{96}Ag lead us to conclude that both $J = 2^+$ and $(2j - 1)$ states should be long-lived.

Acknowledgments

One author (L.Z.) benefited from attending the Weizmann post-NPA5 workshop. He was supported as a visiting professor at the Weizmann Institute by a Morris Belkin award. He thanks Igal Talmi and Michael Kirson for their hospitality. He also thanks Diego Torres for his help in preparing this manuscript and for his critical suggestions.

Appendix A: Interactions discussed in this work

We first show in Table VII the two-body matrix elements that we used in this work in increasing spin from $J = 0$ to $J = J_{\text{max}}$. The even spins have isospin $T = 1$ and the odd ones $T = 0$.

Table VII: Two-body matrix elements in increasing spin from $J = 0$ to $J = J_{\text{max}}$. The even spins have isospin $T = 1$ and the odd ones $T = 0$.

J	$f_{7/2}$		$g_{9/2}$	
	INTa	INTb	INTc	INTd
0	0.0000	0.0000	0.0000	0.0000
1	0.6111	0.5723	1.1387	1.1387
2	1.5863	1.4465	1.3947	1.3947
3	1.4904	1.8224	1.8230	1.8230
4	2.8153	2.6450	2.0823	2.0823
5	1.5101	2.1490	1.9215	1.9215
6	3.2420	2.9600	2.2802	2.2802
7	0.6163	0.1990	1.8797	1.8797
8			2.4275	2.4275
9			1.4964	0.7500

Next we give the two-body matrix elements for the CCGI interaction from $J = 0$ to $J = 9$; they are respectively: -2.317, -1.488, -0.667, -0.440, -0.100, -0.271, 0.066, -0.404, 0.210, -1.402.

Finally, we consider the large scale interactions. In Ref. [6] the KB3 [21] interaction was used in a complete f - p space ($f_{7/2}$, $p_{3/2}$, $f_{5/2}$, $p_{1/2}$) for the study of ^{52}Fe ; in Ref. [7] the GXPF1A [22] interaction was used for the same nucleus and model space. In Ref. [13], to study mainly ^{96}Pd , the SLG [23] and F-FIT [24] interactions were used in the model space ($p_{1/2}$, $g_{9/2}$), together with the JS interaction in a somewhat larger model space

(allowing single-nucleon excitations to the orbitals $g_{7/2}$, $d_{5/2}$, $s_{1/2}$, $d_{3/2}$). Again the model space ($p_{1/2}$, $g_{9/2}$) was used in Ref. [17] for ^{96}Ag with the SLGT interaction [25], while the jj44b interaction [26] was also used but within the model space ($p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$). Finally, in Ref. [18] various interactions were used: GF [28] in the space ($p_{1/2}$, $g_{9/2}$), FPG [29] in ($p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$), and GDS [30] in ($g_{9/2}$, $g_{7/2}$, $d_{5/2}$, $s_{1/2}$, $d_{3/2}$).

-
- [1] A. Escuderos, L. Zamick, and B. F. Bayman, “Wave functions in the $f_{7/2}$ shell, for educational purposes and ideas”, <http://arxiv.org/abs/nuc1-th/0506050> (2005).
 - [2] B.F. Bayman, J.D. McCullen, and L. Zamick, Phys. Rev. Lett. **11**, 215 (1963).
 - [3] J.D. McCullen, B.F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964); Princeton University Technical Report NYO-9891 (1964).
 - [4] J.N. Ginocchio and J.B. French, Phys. Lett. **7**, 137 (1963).
 - [5] L. Zamick, Phys. Atom. Nucl. **65**, 740 (2002).
 - [6] C.A. Ur *et al.*, Phys. Rev. C **58**, 3163 (1998).
 - [7] G. Puddu, Acta Phys. Pol. B **40**, 1685 (2009).
 - [8] National Nuclear Data Center (<http://www.nndc.bnl.gov>).
 - [9] V. Avrigeanu *et al.*, Nucl. Phys. A **272**, 243 (1976).
 - [10] A. Escuderos and L. Zamick, Phys. Rev. C **73**, 044302 (2006).
 - [11] B.S. Nara Singh, Phys. Rev. Lett. **107**, 172502 (2011).
 - [12] K. Schmidt *et al.*, Nucl. Phys. A **624**, 185 (1997).
 - [13] L. Batist *et al.*, Nucl. Phys. A **720**, 245 (2003).
 - [14] R. Grzywacz *et al.*, Phys. Rev. C **55**, 1126 (1997).
 - [15] H. Grawe, A. Blazhev, M. Gorska, R. Grzywacz, H. Mach, and I. Mukha, Eur. Phys. J. A **27**, s01, 257 (2006).
 - [16] C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
 - [17] A.D. Becerril *et al.*, Phys. Rev. C **84**, 041303 (R) (2011).
 - [18] P. Boutachkov *et al.*, Phys. Rev. C **84**, 044311 (2011).
 - [19] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **85**, 034335 (2012).
 - [20] O. Sorlin and M.-G. Porquet, Prog. Part. Nucl. Phys. **61**, 602 (2008).
 - [21] A. Poves and A. Zuker, Phys. Rep. **70**, 235 (1981).
 - [22] M. Honma, T. Otsuka, B.A. Brown, and T. Mizusaki, Eur. Phys. J. A **25**, Suppl. **1**, 499 (2005).
 - [23] F.J.D. Serduke, R.D. Lawson, and D.H. Gloeckner, Nucl. Phys. A **256**, 45 (1976).
 - [24] I.P. Johnstone and L.D. Skouras, Phys. Rev. C **55**, 1227 (1997).
 - [25] H. Herndl and B.A. Brown, Nucl. Phys. A **627**, 35 (1997).
 - [26] B.A. Brown and A.F. Lisetskiy, unpublished. The jj44b Hamiltonian was obtained from a fit to about 600 binding energies and excitation energies with a method similar to that used for the JUN45 Hamiltonian [27].
 - [27] B. Cheal *et al.*, Phys. Rev. Lett. **104**, 252502 (2010).
 - [28] R. Gross and A. Frenkel, Nucl. Phys. A **267**, 85 (1976).
 - [29] T.S. Brock *et al.*, Phys. Rev. C **82**, 061309(R) (2010).
 - [30] R. Machleidt, Phys. Rev. C **63**, 024001 (2001); M. Hjorth-Jensen, T.T.S. Kuo, and E. Osnes, Phys. Rep. **261**, 125 (1995).